

Higher derivative corrections to shear viscosity from graviton's effective coupling

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Higher derivative corrections to shear viscosity from graviton's effective coupling

Nabamita Banerjee^a and Suvankar Dutta^b

^a*Harish-Chandra Research Institute, Allahabad, India*

^b*Dept. of Mathematical and Statistical Science, University of Alberta, Canada*

E-mail: nabamita@mri.ernet.in, sdutta@math.ualberta.ca

ABSTRACT: The shear viscosity coefficient of strongly coupled boundary gauge theory plasma depends on the horizon value of the effective coupling of transverse graviton moving in a black hole background. The proof for the above statement is based on the canonical form of graviton's action. But in presence of generic higher derivative terms in the bulk Lagrangian the action is no longer canonical. We give a procedure to find an effective action for graviton (to first order in coefficient of higher derivative term) in canonical form in presence of any arbitrary higher derivative terms in the bulk. From that effective action we find the effective coupling constant for transverse graviton which in general depends on the radial coordinate r . We also argue that horizon value of this effective coupling is related to the shear viscosity coefficient of the boundary fluid in higher derivative gravity. We explicitly check this procedure for two specific examples: (1) four derivative action and (2) eight derivative action ($Weyl^4$ term). For both cases we show that our results for shear viscosity coefficient (upto first order in coefficient of higher derivative term) completely agree with the existing results in the literature.

KEYWORDS: AdS-CFT Correspondence, Gauge-gravity correspondence

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1 Introduction

The AdS/CFT correspondence is a powerful tool to study different properties of strongly coupled gauge theory in terms of dual (super) gravity theory in AdS space. In low frequency limit the boundary field theory can be described by hydrodynamics. In this limit different transport coefficients like shear viscosity, diffusion constant, thermal and electrical conductivity of strongly coupled boundary fluid have been computed in the context of AdS/CFT (see [1]–[31]).

In [1], the authors evaluated the shear viscosity coefficient of boundary fluid using Kubo formula. This formula relates the shear viscosity to two point function of energy momentum tensor in zero frequency limit. On the other hand from field operator correspondence of

the AdS/CFT conjecture we know that energy momentum tensor of boundary field theory is sourced by bulk graviton excitations. Therefore in the context of AdS/CFT, to calculate thermal two point correlation function of field theory energy momentum tensor we need to add small perturbations to the bulk metric. In [1], the authors considered graviton excitations polarized parallel to the black brane (i.e., xy components are turned on) and moving transverse to it. When one sends the gravitons to the brane, there is a probability that it will be absorbed by the brane. They calculated the absorption coefficient and showed that it is related to two point functions of energy momentum tensor of boundary fluid.

To calculate the absorption coefficients, one needs to solve the wave equation for transverse gravitons. In presence of any higher derivative terms in the bulk action the solution may be technically difficult in general [32–34]. Recently there is a proposal that the shear viscosity of strongly coupled boundary gauge theory plasma is related to the effective coupling of graviton calculated at the black hole horizon [35, 36]. In [37], using the membrane paradigm, the authors have confirmed that at the level of linear response the low frequency limit of strongly coupled boundary field theory at finite temperature is determined by the horizon geometry of its gravity dual. They have proved that generic boundary theory transport coefficients can be expressed in terms of geometric quantities evaluated at the horizon.¹ In particular, they have found that the shear viscosity coefficient is given by the transverse graviton coupling computed at the horizon. The novelty of this result is that one does not need to solve the equation of motion for the graviton to calculate the thermal Green function. From graviton’s action one can easily read off the coupling constant and hence determine the shear viscosity coefficient.

To find the effective coupling of gravitons one has to find the general action. This can be achieved in the following way. Consider the Einstein-Hilbert action with negative cosmological constant

$$I = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} (R + 12) . \tag{1.1}$$

The equation of motion obtained from this action has a black hole solution. We denote this background solution by $g_{\mu\nu}^{(0)}$. Now we consider fluctuation about this spacetime in xy (for example) direction,²

$$g_{xy} = g_{xy}^{(0)} + \epsilon h_{xy}(r, x) = g_{xy}^{(0)} (1 + \epsilon \Phi(r, x)) . \tag{1.2}$$

Then substituting the metric with fluctuation in the action (1.1) and keeping terms up to order ϵ^2 we get the action for graviton. The form of this action is,

$$S \sim \frac{1}{16\pi G_5} \int \frac{d^4k}{(2\pi)^4} dr (a(r)\phi'(r, k)\phi'(r, -k) + b(r)\phi(r, k)\phi(r, -k)) \tag{1.3}$$

where,

$$\phi(r, k) = \int \frac{d^4x}{(2\pi)^4} e^{-ik \cdot x} \Phi(r, x) , \tag{1.4}$$

¹See [38] also.

²Notations: x denotes the boundary coordinates. $x = \{t, \vec{x}\}$.

$k = \{-\omega, \vec{k}\}$ and ‘ ’ denotes derivative with respect to r . The effective coupling is related to the coefficient of ϕ'^2 i.e., a (we have reviewed this calculation in section 2).

This gives the correct viscosity coefficient for the Einstein-Hilbert gravity. But it is not obvious how to generalize this approach for higher derivative case. The proof given in [37] was based on the canonical form (1.3) of graviton’s action. In presence of arbitrary higher derivative terms in the bulk, the general action for the perturbation h_{xy} does not have the above form (1.3). Rather it will have more than two derivative (with respect to r) terms. [37, 39] have considered Gauss-Bonnet term in the bulk action. In general, presence of $R_{ab}R^{ab}$ and $R_{abcd}R^{abcd}$ terms in the bulk result terms like ϕ''^2 and $\phi'\phi''$ in the action for h_{xy} . For Gauss-Bonnet combinations these terms get canceled and the general action still has the form (1.3).

In this paper we have considered generic higher derivatives terms in the bulk Lagrangian. We have given a procedure to construct an effective action S_{eff} for transverse graviton of the form (1.3) in presence of any higher derivative terms in the bulk. The details of the construction is given in section (3). Our construction ensures that in low frequency limit, the calculations of retarded Green function (imaginary part) using either effective action or original action are same. Therefore following the similar argument given in [37], we can relate the shear viscosity coefficient of the boundary fluid with the horizon value of the effective coupling obtained from S_{eff} (section 4). In section (5) we have also discussed how membrane fluid captures the properties of boundary fluid in low frequency limit in generic higher derivative gravity. We have checked our procedure for two cases:

- General four derivative terms, (section (6))
- $Weyl^4$ term which arises in type II string theory (section (7)).

In both examples we get exact agreement between our results and the results that already exist in the literature [33, 34, 40]. Hence we conclude that:

The shear viscosity coefficient of the boundary fluid is given by the horizon value of the effective coupling of transverse graviton obtained from its effective action in presence of arbitrary higher derivative terms in the bulk.

2 Shear viscosity from effective coupling

In this section we briefly review how to calculate the shear viscosity coefficient of the boundary fluid from the effective coupling constant of transverse graviton in Einstein-Hilbert gravity.

We first fix the background spacetime. We start with the following Einstein-Hilbert action in AdS space.

$$I = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} (R + 12) . \tag{2.1}$$

Here we have taken the radius of the AdS space 1. The background spacetime is given by the following metric³

$$ds^2 = -h_t(r)dt^2 + \frac{dr^2}{h_r(r)} + \frac{1}{r}d\vec{x}^2 \quad (2.2)$$

where,

$$h_t(r) = \frac{1-r^2}{r} . \quad (2.3)$$

and

$$h_r(r) = 4r^2(1-r^2) . \quad (2.4)$$

The black hole has horizon at $r_0 = 1$ and the temperature of this black hole is given by,

$$T = \frac{1}{\pi} . \quad (2.5)$$

We consider the following metric perturbation,

$$g_{xy} = g_{xy}^{(0)} + h_{xy}(r, x) = g_{xy}^{(0)}(1 + \epsilon\Phi(r, x)) \quad (2.6)$$

where ϵ is an order counting parameter. We consider terms up to order ϵ^2 in the action of $\Phi(r, x)$. The action (in momentum space) is given by,

$$S = \frac{1}{16\pi G_5} \int \frac{d\omega d^3\vec{k}}{(2\pi)^4} dr \left[\mathcal{A}_{1,1}(r)\phi'(r, -k)\phi'(r, k) \right. \\ \left. + \mathcal{A}_{1,0}(r, k)\phi(r, -k)\phi'(r, k) + \mathcal{A}_{0,0}(r, k)\phi(r, k)\phi(r, -k) \right] \quad (2.7)$$

where, $\mathcal{A}_{i,j}(r, k)$ are functions of r and k and $\phi(r, k)$ is given by (1.4). Up to some total derivative the action (2.7) can be written as footnote Though throughout this paper we have written the four vector k , but in practice we have worked in $\vec{k} \rightarrow 0$ limit. In all the expressions we have dropped the terms proportional to \vec{k} or its power.

$$S = \frac{1}{16\pi G_5} \int \frac{d\omega d^3\vec{k}}{(2\pi)^4} dr \left(\mathcal{A}_1^{(0)}(r)\phi'(r, -k)\phi'(r, k) + \mathcal{A}_0^{(0)}(r, k)\phi(r, k)\phi(r, -k) \right) \quad (2.8)$$

where,

$$\mathcal{A}_1^{(0)}(r) = \frac{r^2 - 1}{r} \quad (2.9)$$

and

$$\mathcal{A}_0^{(0)}(r, k) = \frac{\omega^2}{4r^2(1-r^2)} . \quad (2.10)$$

This can be viewed as an action for minimally coupled scalar field $\phi(r, k)$ with effective coupling given by,

$$K_{\text{eff}}(r) = \frac{1}{16\pi G_5} \frac{\mathcal{A}_1^{(0)}(r)}{\sqrt{-g^{(0)}}g^{rr}} \\ = \frac{1}{16\pi G_5} \left(-\frac{1}{2} \right) . \quad (2.11)$$

³We are working in a coordinate frame where asymptotic boundary is at $r \rightarrow 0$.

Therefore according to [37, 39] the effective coupling K_{eff} calculated at the horizon r_0 gives the shear viscosity coefficient of boundary fluid,

$$\begin{aligned} \eta &= r_0^{-\frac{3}{2}}(-2K_{\text{eff}}(r_0)) \\ &= \frac{1}{16\pi G_5} . \end{aligned} \tag{2.12}$$

Therefore shear viscosity to entropy density ratio is given by,

$$\frac{\eta}{s} = \frac{1}{4\pi} . \tag{2.13}$$

The universality of this result is an artifact of the universality of the transverse graviton's effective coupling (2.11) in two derivative gravity.

3 The effective action

Having understood the above procedure to determine the shear viscosity coefficient from the effective coupling of transverse graviton it is tempting to generalize this method for any higher derivative gravity. As we discussed in the introduction, the first problem one faces is that the action for transverse graviton no more has the canonical form (2.7). For generic 'n' derivative gravity theory the action can have terms with (and up to) 'n' derivatives of $\Phi(r, x)$. Therefore, from that action it is not very clear how to determine the effective coupling. In this section we try to address this issue.

We construct an effective action which is of form (2.8) with different coefficients capturing higher derivative effects. We determine these two coefficients by claiming that the equation of motion for $\phi(r, k)$ coming from these two actions (general action and effective action) are same up to first order in perturbation expansion (in coefficient of higher derivative term). Once we determine the effective action for transverse graviton in canonical form then we can extract the effective coupling from the coefficient of $\phi'(r, k)\phi'(r, -k)$ term in the action. Needless to say, our method is perturbatively correct.

3.1 The general action and equation of motion

Let us start with a generic 'n' derivative term in the action with coefficient μ . We study this system perturbatively and all our expressions are valid up to order μ . The action is given by,

$$S = \frac{1}{16\pi G_5} \int d^5x \left(R + 12 + \mu \mathcal{R}^{(n)} \right) \tag{3.1}$$

where, $\mathcal{R}^{(n)}$ is any n derivative Lagrangian. The metric in general is given by (assuming planar symmetry),

$$ds^2 = -(h_t(r) + \mu h_t^{(n)}(r))dt^2 + \frac{dr^2}{h_r(r) + \mu h_r^{(n)}(r)} + \frac{1}{r}(1 + \mu h_s^{(n)}(r))d\vec{x}^2 \tag{3.2}$$

where $h_t^{(n)}, h_r^{(n)}$ and $h_s^{(n)}$ are higher derivative corrections to the metric.

Substituting the background metric with fluctuations in the action (3.1) (we call it general action or original action) for the scalar field $\phi(r, k)$ we get,

$$S = \frac{1}{16\pi G_5} \int \frac{d^4k}{(2\pi)^4} dr \sum_{p,q=0}^n \mathcal{A}_{p,q}(r, k) \phi^{(p)}(r, -k) \phi^{(q)}(r, k) \quad (3.3)$$

where, $\phi^{(p)}(r, k)$ denotes the p^{th} derivative of the field $\phi(r, k)$ with respect to r and $p+q \leq n$. The coefficients $\mathcal{A}_{p,q}(r, k)$ in general depends on the coupling constant μ . $\mathcal{A}_{p,q}$ with $p+q \geq 3$ are proportional to μ and vanishes in $\mu \rightarrow 0$ limit, since the terms $\phi^{(p)}\phi^{(q)}$ with $p+q \geq 3$ appears as an effect of higher derivative terms in the action (3.1). Up to some total derivative terms, the general action (3.3) can also be written as,

$$\begin{aligned} S &= \frac{1}{16\pi G_5} \int \frac{d^4k}{(2\pi)^4} dr \sum_{p=0}^{n/2} \mathcal{A}_p(r, k) \phi^{(p)}(r, -k) \phi^{(p)}(r, k), \quad n \text{ even} \\ &= \frac{1}{16\pi G_5} \int \frac{d^4k}{(2\pi)^4} dr \sum_{p=0}^{(n-1)/2} \mathcal{A}_p(r, k) \phi^{(p)}(r, -k) \phi^{(p)}(r, k), \quad n \text{ odd}. \end{aligned} \quad (3.4)$$

The equation of motion for the scalar field $\phi(r, k)$ is given by,

$$\begin{aligned} \sum_{p=0}^{n/2} \left(-\frac{d}{dr} \right)^p \frac{\partial \mathcal{L}(\{\phi^{(m)}\})}{\partial \phi^{(p)}(r, k)} &= 0, \quad n \text{ even} \\ \sum_{p=0}^{(n-1)/2} \left(-\frac{d}{dr} \right)^p \frac{\partial \mathcal{L}(\{\phi^{(m)}\})}{\partial \phi^{(p)}(r, k)} &= 0, \quad n \text{ odd} \end{aligned} \quad (3.5)$$

where $\mathcal{L}(\{\phi^{(m)}\})$ is given by

$$\mathcal{L}(\{\phi^{(m)}\}) = \sum_p \mathcal{A}_p(r, k) \phi^{(p)}(r, -k) \phi^{(p)}(r, k). \quad (3.6)$$

We analyze the general action for the scalar field $\phi(r, k)$ and their equation of motion perturbatively and write an effective action for the field $\phi(r, k)$.

The generic form of the equation of motion (varying the general action) upto order μ is given by,

$$\mathcal{A}_0(r, k) \phi(r, k) - \mathcal{A}'_1(r, k) \phi'(r, k) - \mathcal{A}_1(r, k) \phi''(r, k) = \mu \hat{\mathcal{F}}(\{\phi^{(p)}\}) + \mathcal{O}(\mu^2) \quad (3.7)$$

where $\hat{\mathcal{F}}(\{\phi^{(p)}\})$ is some linear function of double and higher derivatives of $\phi(r, k)$, coming from two or higher derivative terms in action (3.3). The zeroth order ($\mu \rightarrow 0$) equation of motion is given by,

$$\mathcal{A}_0^{(0)}(r, k) \phi(r, k) - \mathcal{A}'_1{}^{(0)}(r, k) \phi'(r, k) - \mathcal{A}_1^{(0)}(r, k) \phi''(r, k) = 0 \quad (3.8)$$

where, $\mathcal{A}_p^{(0)}$ is the value of \mathcal{A}_p at $\mu \rightarrow 0$. From this equation we can write $\phi''(r, k)$ in terms of $\phi'(r, k)$ and $\phi(r, k)$ in $\mu \rightarrow 0$ limit.

$$\phi''(r, k) = \frac{\mathcal{A}_0^{(0)}(r, k)}{\mathcal{A}_1^{(0)}(r, k)} \phi(r, k) - \frac{\mathcal{A}'_1{}^{(0)}(r, k)}{\mathcal{A}_1^{(0)}(r, k)} \phi'(r, k). \quad (3.9)$$

Then the full equation of motion can be written in the following way,

$$\mathcal{A}_0^{(0)}(r, k)\phi(r, k) - \mathcal{A}_1^{\prime(0)}(r, k)\phi'(r, k) - \mathcal{A}_1^{(0)}(r, k)\phi''(r, k) = \mu \tilde{\mathcal{F}}(\phi(r, k), \phi'(r, k), \phi''(r, k), \dots) + \mathcal{O}(\mu^2). \quad (3.10)$$

Since the right hand side of equation (3.10) is proportional to μ , we can replace the $\phi''(r, k)$ and other higher (greater than 2) derivatives of $\phi(r, k)$ by its leading order value (3.9). Therefore up to order μ the equation of motion for ϕ is given by,

$$\begin{aligned} \mathcal{A}_0^{(0)}(r, k)\phi(r, k) - \mathcal{A}_1^{\prime(0)}(r, k)\phi'(r, k) - \mathcal{A}_1^{(0)}(r, k)\phi''(r, k) &= \mu \mathcal{F}(\phi(r, k), \phi'(r, k)) \\ &+ \mathcal{O}(\mu^2) \\ &= \mu(\mathcal{F}_1\phi'(r, k) + \mathcal{F}_0\phi(r, k)) \\ &+ \mathcal{O}(\mu^2) \end{aligned} \quad (3.11)$$

where \mathcal{F}_0 and \mathcal{F}_1 are some function of r . This is the perturbative equation of motion for the scalar field $\phi(r, k)$ obtained from the general action (3.3).

3.2 Strategy to find the effective action

In this subsection we describe the strategy to write an effective action for the field $\phi(r, k)$ which has form (2.8) with different functions. The prescription is following.

(a) We demand the equation of motion for $\phi(r, k)$ obtained from the original action and the effective action are same upto order μ . This will fix the coefficients of ϕ'^2 and ϕ^2 terms in effective action.

Let us start with the following form of the effective action.

$$S_{\text{eff}} = \frac{1}{16\pi G_5} \int \frac{d\omega d^3\vec{k}}{(2\pi)^4} dr \left[(\mathcal{A}_1^{(0)}(r, k) + \mu\mathcal{B}_1(r, k))\phi'(r, -k)\phi'(r, k) + (\mathcal{A}_0^{(0)}(r, k) + \mu\mathcal{B}_0(r, k))\phi(r, k)\phi(r, -k) \right]. \quad (3.12)$$

The functions \mathcal{B}_0 and \mathcal{B}_1 are yet to be determined. We determine these functions by claiming that the equation of motion for the scalar field $\phi(r, k)$ obtained from this effective action is same as (3.11) up to order μ . The equation of motion for $\phi(r, k)$ from the effective action is given by,

$$\begin{aligned} \mathcal{A}_0^{(0)}(r, k)\phi(r, k) - \mathcal{A}_1^{\prime(0)}(r, k)\phi'(r, k) - \mathcal{A}_1^{(0)}(r, k)\phi''(r, k) \\ = \mu \left(\mathcal{B}'_1(r, k) - \frac{\mathcal{A}'_1{}^{(0)}(r, k)}{\mathcal{A}_1^{(0)}(r, k)}\mathcal{B}_1(r, k) \right) \phi'(r, k) \\ + \mu \left(\mathcal{B}_1(r, k)\frac{\mathcal{A}_0^{(0)}(r, k)}{\mathcal{A}_1^{(0)}(r, k)} - \mathcal{B}_0(r, k) \right) \phi(r, k) + \mathcal{O}(\mu^2). \end{aligned} \quad (3.13)$$

Therefore comparing with (3.11) we get,

$$\mathcal{B}'_1(r, k) - \frac{\mathcal{A}'_1{}^{(0)}(r, k)}{\mathcal{A}_1^{(0)}(r, k)}\mathcal{B}_1(r, k) - \mathcal{F}_1(r, k) = 0 \quad (3.14)$$

and

$$\mathcal{B}_0(r, k) = \mathcal{B}_1(r, k) \frac{\mathcal{A}_0^{(0)}(r, k)}{\mathcal{A}_1^{(0)}(r, k)} - \mathcal{F}_0(r, k) . \quad (3.15)$$

The solutions are given by,

$$\begin{aligned} \mathcal{B}_1(r, k) &= \mathcal{A}_1^{(0)}(r, k) \int dr \frac{\mathcal{F}_1(r, k)}{\mathcal{A}_1^{(0)}(r, k)} + \kappa \mathcal{A}_1^{(0)}(r, k) \\ &= \tilde{\mathcal{B}}_1(r, k) + \kappa \mathcal{A}_1^{(0)}(r, k) \end{aligned} \quad (3.16)$$

and

$$\mathcal{B}_0 = \tilde{\mathcal{B}}_0(r, k) + \kappa \mathcal{A}_0^{(0)} \quad (3.17)$$

for some constant κ . We need to fix this constant.

(b) Condition **(a)** can not fix the overall normalization factor of the effective action. In particular we can multiply it by $(1 + \mu\Gamma)$ (for some constant Γ) and still get the same equation of motion.⁴ Considering this normalization, the effective action is given by,

$$\begin{aligned} S_{\text{eff}} = \frac{1 + \mu \Gamma}{16\pi G_5} \int \frac{d\omega d^3 \vec{k}}{(2\pi)^4} dr \left[(\mathcal{A}_1^{(0)}(r, k) + \mu \mathcal{B}_1(r, k)) \phi'(r, -k) \phi'(r, k) \right. \\ \left. + (\mathcal{A}_0^{(0)}(r, k) + \mu \mathcal{B}_0(r, k)) \phi(r, k) \phi(r, -k) \right] . \end{aligned} \quad (3.18)$$

Substituting the values of \mathcal{B} 's (3.16) and (3.17) we get,

$$S_{\text{eff}} = (1 + \mu(\Gamma + \kappa)) S^{(0)} + \mu \int dr \left(\tilde{\mathcal{B}}_1(r, k) \phi'(r, -k) \phi'(r, k) + \tilde{\mathcal{B}}_0(r, k) \phi(r, -k) \phi(r, k) \right) \quad (3.19)$$

where $S^{(0)}$ is the effective action at $\mu \rightarrow 0$ limit. This implies that the integration constant κ can be absorbed in the overall normalization constant Γ . Henceforth we will denote this combination as Γ .

Our prescription is to take Γ to be **zero** from the following observation.

The shear viscosity coefficient of boundary fluid is related to the imaginary part of retarded Green function in low frequency limit. The retarded Green function $G_{xy,xy}^R(k)$ is defined in the following way. The on-shell action for graviton can be written as a surface term,

$$S \sim \int \frac{d^4 k}{(2\pi)^4} \phi_0(k) \mathcal{G}_{xy,xy}(k, r) \phi_0(-k) \quad (3.20)$$

where $\phi_0(k)$ is the boundary value of $\phi(r, k)$ and $G_{xy,xy}^R$ is given by,

$$G_{xy,xy}^R(k) = \lim_{r \rightarrow 0} 2\mathcal{G}_{xy,xy}(k, r) \quad (3.21)$$

⁴We are thankful to Ashoke Sen for raising this point.

and shear viscosity coefficient is given by,⁵

$$\eta = \lim_{\omega \rightarrow 0} \left[\frac{1}{\omega} \text{Im} G_{xy,xy}^R(k) \right] \quad (\text{computed on-shell}). \quad (3.22)$$

Now it turns out that the imaginary part of this retarded Green function obtained from the original action and effective action are same upto the normalization constant Γ in presence of generic higher derivative terms in the bulk action. Therefore it is quite natural to take Γ to be *zero* as it ensures that starting from the effective action also one can get same shear viscosity using Kubo machinery. To show that the above statement is true we do not need to know the full solution for ϕ , in other words to find the difference between the two Green functions one does not need to calculate the Green functions explicitly. Assuming the following general form of solution for ϕ

$$\phi \sim (1 - r^2)^{-i\omega\beta} (1 + i\omega\beta\mu\xi(r)) \quad (3.23)$$

it can be shown generically.⁶ In appendix A we have given the proof.

Because of the canonical form of the effective action, it follows from the argument in [37] and the statement above, that the shear viscosity coefficient of boundary fluid is given by the horizon value of the effective coupling obtained from the effective action in presence of any higher derivative terms in the bulk action. We discuss elaborately on this point in section (4).

(c) After getting the effective action for $\phi(r, k)$, the effective coupling is given by,

$$K_{\text{eff}}(r) = \frac{1}{16\pi G_5} \frac{\mathcal{A}_1^{(0)}(r, k) + \mu\mathcal{B}_1(r, k)}{\sqrt{-g}g^{rr}} \quad (3.24)$$

where g^{rr} is the 'rr' component of the inverse perturbed metric and $\sqrt{-g}$ is the determinant of the perturbed metric. Hence the shear viscosity coefficient is given by,

$$\eta = r_0^{-\frac{3}{2}} (-2K_{\text{eff}}(r = r_0)) \quad (3.25)$$

where r_0 is the corrected horizon radius.

To summaries, we have obtained a well defined procedure to find the correction (up to order μ) to the coefficient of shear viscosity of the boundary fluid in presence of general higher derivative terms in the action.

4 Flow from boundary to horizon

Following [37], let us define the following linear response function

$$\bar{\chi}(r, k) = \frac{\Pi(r, k)}{i\omega\phi(r, k)} \quad (4.1)$$

⁵To calculate this number one has to know the exact solution, i.e., the form of ξ and the value of β in (3.23).

⁶In fact one advantage to calculate the boundary term from effective action is that we do not need to consider the *Gibbons-Hawking* boundary terms (see appendix A).

where $\Pi(r, k)$ is conjugate momentum of the scalar field ϕ (with respect to a foliation in the r direction),

$$\begin{aligned}\Pi(r, k) &= \left(\mathcal{A}_1^{(0)}(r, k) + \mu \mathcal{B}_1(r, k) \right) \phi'(r, -k) \\ &= \tilde{K}_{\text{eff}}(r) \sqrt{-g^{(0)}} g^{(0)rr} \partial_r \phi\end{aligned}\quad (4.2)$$

where $\tilde{K}_{\text{eff}}(r) = 16\pi G_5 K_{\text{eff}}(r)$. Now we will show, using the equation of motion, that the function $\Pi(r, k)$ and the combination $\omega\phi(r, k)$ is independent of the radial coordinate r in $k \rightarrow 0$ limit. The equation of motion is given by,

$$\begin{aligned}\frac{d}{dr} \left[\left(\mathcal{A}_1^{(0)}(r, k) + \mu \mathcal{B}_1(r, k) \right) \phi'(r, k) \right] &= \left(\mathcal{A}_0^{(0)}(r, k) + \mu \mathcal{B}_0(r, k) \right) \phi(r, k) \\ \frac{d}{dr} \left[\Pi(r, k) \right] &= \left(\mathcal{A}_0^{(0)}(r, k) + \mu \mathcal{B}_0(r, k) \right) \phi(r, k).\end{aligned}\quad (4.3)$$

Since $\mathcal{A}_0^{(0)} \sim \omega^2$, therefore it follows from (4.3) and (4.2) that, in $\mu \rightarrow 0$ limit $\Pi(r, k)$ and $\omega\phi(r, k)$ are independent of r . But this is true even in $\mu \neq 0$ case. To understand this we note that, function \mathcal{A}_0 in (3.4) is proportional to ω^2 in general.⁷ Therefore it follows from (3.9), (3.11) and (3.15) that \mathcal{B}_0 is also proportional to ω^2 . Hence, in presence of higher derivative terms also it follows from (4.2) and (4.3) that the function $\Pi(r, k)$ and $\omega\phi(r, k)$ are independent of radial direction r in low frequency limit.

Therefore this response function calculated at the asymptotic boundary and at the horizon gives the same result and is equal to the shear viscosity coefficient. One can calculate the function $\tilde{\chi}$ and it turns out that,

$$\begin{aligned}\tilde{\chi}(r=0, k \rightarrow 0) &= \frac{\text{Im} G_{xy,xy}^{\text{eff}}}{i\omega}, \\ \tilde{\chi}(r=r_0, k \rightarrow 0) &= -\frac{r_0^{-3/2}}{8\pi G_5} \frac{\mathcal{A}_1^{(0)}(r, k) + \mu \mathcal{B}_1(r, k)}{\sqrt{-g g^{rr}}} \Big|_{r_0} = r_0^{-\frac{3}{2}} (-2K_{\text{eff}}(r_0)).\end{aligned}\quad (4.4)$$

Thus, shear viscosity coefficient of boundary fluid is related to horizon value of graviton's effective coupling obtained from the effective action.

5 Membrane fluid in higher derivative gravity

The **UV/IR** connection tells us that the boundary field theory physics in low frequency limit should be governed by the near horizon geometry of its gravity dual. In [37], the authors have established a connection between horizon membrane fluid and boundary fluid in linear response approximation. They considered a mass less scalar field (with action given in (2.8)) outside the horizon and studied the response of the membrane fluid to this bulk scalar field. They defined a *membrane charge* Π_{mb} which is equal to the conjugate

⁷In general when we write action (3.4) action (3.3) we get some terms like $\omega^2\phi^2 + Z(r)\phi^2$. The function $Z(r)$ is zero when background equation of motion is satisfied. We have explicitly checked this for two, four and eight derivative case.

momentum of the scalar field ϕ (with respect to a foliation in the r direction) at the horizon. Imposing regularity condition on the scalar field at the horizon they interpreted the membrane charge Π_{mb} as a response of the horizon fluid to the scalar field. Considering the scalar field ϕ to be bulk graviton excitation (h_x^y), Π_{mb} gives the shear viscosity of the membrane (horizon) fluid which is also equal to horizon value of the effective coupling of graviton. In this way, they proved that the shear viscosity of boundary fluid is related to that of membrane fluid.

In higher derivative gravity, since the canonical form of the action (2.8) breaks down, it is not very obvious how to define the membrane charge Π_{mb} . Instead of the original action if we consider the effective action (3.12) for graviton then it is possible to write the membrane action perturbatively and define the membrane charge (Π_{mb}) in higher derivative gravity. As if the membrane fluid is sensitive to the effective action S_{eff} in higher derivative gravity.

Following [37] we can write the membrane action and charge in the following way (in momentum space)

$$S_{\text{mb}} = \int_{\Sigma} \frac{d^4k}{(2\pi)^4} \sqrt{-\sigma} \left(\frac{\Pi(r_0, k)}{\sqrt{-\sigma}} \phi(r_0, -k) \right) \quad (5.1)$$

where $\sigma_{\mu\nu}$ is the induced metric on the membrane and $\Pi(r, k)$ is given by (4.2) and the membrane charge is given by,

$$\Pi_{\text{mb}} = \frac{\Pi(r_0, k)}{\sqrt{-\sigma}} = -\tilde{K}_{\text{eff}}(r_0) \sqrt{g^{(0)rr}} \partial_r \phi(r, k) \Big|_{r_0} . \quad (5.2)$$

Imposing the in-falling wave boundary condition on ϕ , it can be shown that the membrane charge Π_{mb} is the response of the horizon fluid to the bulk graviton excitation and the membrane fluid transport coefficient is given by,

$$\eta_{\text{mb}} = \tilde{K}_{\text{eff}}(r_0) . \quad (5.3)$$

Hence, we see that even in higher derivative gravity the shear viscosity coefficient of boundary fluid is captured by the membrane fluid.

6 Four derivative lagrangian

In this section we apply our effective action approach to calculate the correction to the shear viscosity in presence of general four derivative terms in the action. The four derivative bulk action we consider is of the following form

$$S = \frac{1}{16\pi G_5} \int d^5x \left[R + 12 + \mu \left(c_1 R^2 + c_2 R_{ab} R^{ab} + c_3 R_{abcd} R^{abcd} \right) \right] \quad (6.1)$$

with constant c_1 , c_2 and c_3 . The background metric is given by,

$$ds^2 = -\frac{f(r)}{r} dt^2 + \frac{dr^2}{4r^2 f(r)} + \frac{1}{r} d\vec{x}^2 \quad (6.2)$$

where,

$$f(r) = 1 - r^2 + \frac{\mu}{3} (4(5c_1 + c_2) + 2c_3) + 2\mu c_3 r^4 . \quad (6.3)$$

The position of the horizon is given by,

$$f(r_0) = 0 \quad (6.4)$$

which implies that,

$$r_0 = 1 + \frac{2}{3}(5c_1 + c_2 + 2c_3)\mu + \mathcal{O}(\mu^2) . \quad (6.5)$$

The temperature of this black hole is given by,

$$T = \frac{1}{\pi} + \frac{(5c_1 + c_2 - 7c_3)\mu}{3\pi} + \mathcal{O}(\mu^2) . \quad (6.6)$$

In this coordinate frame the boundary metric is given by,

$$ds_4^2 = (-f(0)dt^2 + d\vec{x}^2) \quad (6.7)$$

which is not Minkowskian. Therefore we rescale our time coordinate to make the boundary metric Minkowskian. We replace,

$$t \rightarrow \frac{t}{\sqrt{f(0)}} \quad (6.8)$$

in the metric (6.2). The rescaled metric is,

$$ds^2 = -\frac{f(r)}{f(0)r}dt^2 + \frac{dr^2}{4r^2f(r)} + \frac{1}{r}d\vec{x}^2 . \quad (6.9)$$

This is our background metric and we consider fluctuation around this.

6.1 The general action

In this theory, the general action for the scalar field $\phi(r, k)$ is given by,

$$S = \frac{1}{16\pi G_5} \int \frac{d^4k}{(2\pi)^4} dr \left[A_1^{GB}(r, k)\phi(r, k)\phi(r, -k) + A_2^{GB}(r, k)\phi'(r, k)\phi'(r, -k) \right. \quad (6.10) \\ \left. + A_3^{GB}(r, k)\phi''(r, k)\phi''(r, -k) + A_4^{GB}(r, k)\phi(r, k)\phi'(r, -k) \right. \\ \left. + A_5^{GB}(r, k)\phi(r, k)\phi''(r, -k) + A_6^{GB}(r, k)\phi'(r, k)\phi''(r, -k) \right]$$

where the expressions for A_i^{GB} s are given in appendix B. Up to some total derivative terms this action can be written as,

$$S = \frac{1}{16\pi G_5} \int \frac{d^4k}{(2\pi)^4} dr \left[\mathcal{A}_0^{GB}\phi(r, k)\phi(r, -k) \right. \quad (6.11) \\ \left. + \mathcal{A}_1^{GB}\phi'(r, k)\phi'(r, -k) + \mathcal{A}_2^{GB}\phi''(r, k)\phi''(r, -k) \right]$$

where,

$$\begin{aligned} \mathcal{A}_0^{GB} &= A_1^{GB}(r, k) - \frac{A_4'^{GB}(r, k)}{2} + \frac{A_5''^{GB}(r, k)}{2} \\ \mathcal{A}_1^{GB} &= A_2^{GB}(r, k) - A_5^{GB}(r, k) - \frac{A_6'^{GB}(r, k)}{2} \\ \mathcal{A}_2^{GB} &= A_3^{GB}(r, k) . \end{aligned} \quad (6.12)$$

6.2 The effective action and shear viscosity

Following the general discussion of section (3) we write the effective action for the scalar field,

$$S_{\text{eff}}^{GB} = \frac{(1 + \Gamma\mu)}{16\pi G_5} \int \frac{d^4 k}{(2\pi)^4} \left[(\mathcal{A}_1^{(0)}(r, k) + \mu \mathcal{B}_1^{GB}(r, k)) \phi'(r, -k) \phi'(r, k) \right. \\ \left. + (\mathcal{A}_0^{(0)}(r, k) + \mu \mathcal{B}_0^{GB}(r, k)) \phi(r, k) \phi(r, -k) \right].$$

To evaluate the functions \mathcal{B}_1^{GB} and \mathcal{B}_0^{GB} and to fix the normalization constant Γ , we follow the strategy given in section (3.2). Comparing the equation of motion for $\phi(r, k)$ from two actions we get the function \mathcal{B}_1^{GB} and \mathcal{B}_0^{GB} of the following form,

$$\mathcal{B}_0^{GB} = \frac{\omega^2}{12r^2(1-r^2)^2} (10(11r^2 - 13)c_1 + (22r^2 - 26)c_2 + (11 - 25r^2 + 6r^4)c_3) \\ \mathcal{B}_1^{GB} = \frac{1}{3r} ((110 - 130r^2)c_1 + (22 - 26r^2)c_2 - (13 - 23r^2 + 18r^4)c_3). \quad (6.13)$$

The normalization constant $\Gamma = 0$ (appendix A).

Now we can calculate the effective coupling using the formula (3.24). It turns out to be,

$$K_{\text{eff}}(r) = \frac{1}{16\pi G_5} \left(-\frac{1}{2} + (4(5c_1 + c_2) - 2(1 - r^2)c_3) \mu \right). \quad (6.14)$$

Therefore the shear viscosity is given by,

$$\eta = \frac{1}{r_0^{3/2}} (-2K_{\text{eff}}(r_0)) \\ = \frac{1}{16\pi G_5} \frac{1}{r_0^{3/2}} (1 - 8(5c_1 + c_2)\mu) \\ = \frac{1}{16\pi G_5} (1 - 9\mu(5c_1 + c_2) - 2\mu c_3). \quad (6.15)$$

This result is in agreement with [33, 34, 41].

7 String theory correction to shear viscosity

In this section we apply the effective action approach for eight derivative terms in the Lagrangian. We consider the well known *Weyl*⁴ term. This term appears in type II string theory. The five dimensional bulk action is given by,

$$S = \frac{1}{16\pi G_5} \int d^5 x \sqrt{-g} (R + 12 + \mu W^{(4)}) \quad (7.1)$$

where,

$$W^{(4)} = C^{hmnk} C_{pmnq} C_h^{rsp} C_{rsk}^q + \frac{1}{2} C^{hkmn} C_{pqmn} C_h^{rsp} C_{rsk}^q \quad (7.2)$$

and the weyl tensors C_{abcd} are given by,

$$C_{abcd} = R_{abcd} + \frac{1}{3}(g_{ad}R_{cb} + g_{bc}R_{ad} - g_{ac}R_{db} - g_{bd}R_{ca}) + \frac{1}{12}(g_{ac}g_{bd} - g_{ad}g_{cb})R . \quad (7.3)$$

The background metric is given by [42, 43],

$$ds^2 = -\frac{(1-r^2)}{r} (1 + 45\mu r^6 - 75\mu r^4 - 75\mu r^2) dt^2 + \frac{1}{4(1-r^2)r^2} (1 - 285\mu r^6 + 75\mu r^4 + 75\mu r^2) dr^2 + \frac{1}{r} d\vec{x}^2 . \quad (7.4)$$

The temperature of this black hole is given by,

$$T = \frac{1}{\pi} (1 + 15\mu) . \quad (7.5)$$

The horizon is located at $r_0 = 1$.

7.1 The general action

Putting the perturbed metric in (7.1) we get the general action for the scalar field $\phi(r, k)$,

$$S = \frac{1}{16\pi G_5} \int \frac{d^4k}{(2\pi)^4} dr \left[A_1^W(r, k) \phi(r, k) \phi(r, -k) + A_2^W(r, k) \phi'(r, k) \phi'(r, -k) \right. \\ \left. + A_3^W(r, k) \phi''(r, k) \phi''(r, -k) + A_4^W(r, k) \phi(r, k) \phi'(r, -k) \right. \\ \left. + A_5^W(r, k) \phi(r, k) \phi''(r, -k) + A_6^W(r, k) \phi'(r, k) \phi''(r, -k) \right] . \quad (7.6)$$

The coefficients A_i^W s are given in appendix (C). Like four derivative case, up to some total derivative terms this action can be written as,

$$S = \frac{1}{16\pi G_5} \int \frac{d^4k}{(2\pi)^4} dr \left[\mathcal{A}_0^W \phi(r, k) \phi(r, -k) + \mathcal{A}_1^W \phi'(r, k) \phi'(r, -k) + \mathcal{A}_2^W \phi''(r, k) \phi''(r, -k) \right] \quad (7.7)$$

where,

$$\mathcal{A}_0^W = A_1^W(r, k) - \frac{A_4'^W(r, k)}{2} + \frac{A_5''^W(r, k)}{2} \\ \mathcal{A}_1^W = A_2^W(r, k) - A_5^W(r, k) - \frac{A_6'^W(r, k)}{2} \\ \mathcal{A}_2^W = A_3^W(r, k) . \quad (7.8)$$

7.2 The effective action and shear viscosity

We write the effective action for the scalar field in the following way,

$$S_{\text{eff}}^W = \frac{(1 + \Gamma\mu)}{16\pi G_5} \int \frac{d^4k}{(2\pi)^4} \left[(\mathcal{A}_1^{(0)}(r, k) + \mu \mathcal{B}_1^W(r, k)) \phi'(r, -k) \phi'(r, k) \right. \\ \left. + (\mathcal{A}_0^{(0)}(r, k) + \mu \mathcal{B}_0^W(r, k)) \phi(r, k) \phi(r, -k) \right] . \quad (7.9)$$

The functions \mathcal{B}_0^W and \mathcal{B}_1^W are given by,

$$\mathcal{B}_0^W(r, k) = -\frac{\omega^2 (663r^6 - 573r^4 + 75r^2)}{4r^2 (r^2 - 1)} \quad (7.10)$$

$$\mathcal{B}_1^W(r, k) = \frac{(r^2 - 1) (129r^6 + 141r^4 - 75r^2)}{r} . \quad (7.11)$$

The normalization constant $\Gamma = 0$ (appendix A).

The effective coupling constant is given by (3.24),

$$\begin{aligned} K_{\text{eff}}(r) &= \frac{1}{16\pi G_5} \frac{\mathcal{A}_1^{(0)}(r, k) + \mu \mathcal{B}_1^W(r, k)}{\sqrt{-g}g^{rr}} \\ &= \frac{1}{16\pi G_5} \left(-\frac{1}{2} (1 + 36\mu r^4(6 - r^2)) \right) . \end{aligned} \quad (7.12)$$

Therefore the shear viscosity is given by,

$$\begin{aligned} \eta &= r_0^{-\frac{3}{2}} (-2K_{\text{eff}}(r_0)) \\ &= \frac{1}{16\pi G_5} (1 + 180\mu), \quad (r_0 = 1) \end{aligned} \quad (7.13)$$

and shear viscosity to entropy density ratio

$$\frac{\eta}{s} = \frac{1}{4\pi} (1 + 120\mu) \quad (7.14)$$

where entropy density s is given by [42, 43],

$$s = \frac{1}{4G_5} (1 + 60\mu) . \quad (7.15)$$

These results agree with [40].⁸

8 Discussion

We have found a procedure to construct an effective action for the transverse graviton in canonical form in presence of any higher derivative terms in bulk and showed that the horizon value of the effective coupling obtained from the effective action gives the shear viscosity coefficient of boundary fluid. Our results are valid upto first order in μ . We discussed two non trivial examples to check the method. We have considered four derivative and eight derivative (*Weyl*⁴) Lagrangian and calculated the correction to the shear viscosity using our method. We found complete agreement between our result and the results obtained using other methods.

Since the equation of motion for scalar field $\phi(r, k)$ obtained from effective and original actions are same, these two actions should be related by some field re-definition. If one finds such field re-definition then the normalization of the effective action will be fixed automatically.⁹

⁸In fact, in [32] the result for η/s was not correct. Later the author(s) corrected their results in [40].

⁹We are thankful to Ashoke Sen for discussion on thins point.

In [35] the authors have proposed a formula for shear viscosity for generalized higher derivative gravity in terms of some geometric quantity evaluated at the event horizon (like Wald's formula for entropy). Though their proposal gives correct results for Einstein-Hilbert and Gauss-Bonnet action but unfortunately we are unable to get the correct result for $Weyl^4$ term. We find the shear viscosity coefficient for $Weyl^4$ term (using their proposal)

$$\eta = \frac{1}{16\pi G_5} (1 + 20\mu) \tag{8.1}$$

which implies,

$$\frac{\eta}{s} = \frac{1}{4\pi} (1 - 40\mu) . \tag{8.2}$$

These issues are under investigation [44].

In this paper we have concentrated on a particular transport coefficient, namely the shear viscosity coefficient. But the other transport coefficients like electrical and thermal conductivity of the boundary fluid can also be captured in terms of the membrane fluid. It would also be interesting to study these other transport coefficients in the context of higher derivative gravity.

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A Fixing the normalization constant

In this appendix we fix the normalization constant Γ . We consider a general class of action for ϕ which appears when the higher derivative terms are made of different contraction of Ricci tensor, Riemann tensor, Weyl tensor, Ricci scalar etc. or their different powers. Since, all these tensors involve two derivatives of metric they can only have terms like $\partial_a \partial_b \Phi(r, x)$ and its lower derivatives. Therefor the most generic quadratic (in $\Phi(r, x)$, in linear response theory) action for this kind of higher derivative gravity has the following form (in momentum space)¹⁰

$$S = \frac{1}{16\pi G_5} \int \frac{d^4 k}{(2\pi)^4} dr \left[a1(r)\phi(r)^2 + a2(r)\phi'(r)^2 + a4(r)\phi(r)\phi'(r) \right. \\ \left. + \mu a6(r)\phi''(r)\phi'(r) + \mu a3(r)\phi''(r)^2 + a5(r)\phi(r)\phi''(r) \right] \tag{A.1}$$

¹⁰In all the expressions we have omitted k dependence of ϕ .

where,

$$\begin{aligned}
 a1(r) &= \frac{-8r^2 + \omega^2 r + 8}{4r^3 - 4r^5} + \mu f2(r) \\
 a2(r) &= -3r + \frac{3}{r} + \mu h2(r) \\
 a4(r) &= -\frac{6}{r^2} - 2 + \mu g2(r) \\
 a5(r) &= -4r + \frac{4}{r} + \mu j2(r)
 \end{aligned} \tag{A.2}$$

and $a3(r), a6(r), j2(r), g2(r), h2(r)$ and $f2(r)$ depends on higher derivative terms in the action. Now let us write the effective Lagrangian as follows,

$$S_{\text{eff}} = \frac{1 + \mu\Gamma}{16\pi G_5} \int \frac{d^4 k}{(2\pi)^4} dr \left[\frac{4r (r^2 - 1)^2 \phi'(r)^2 - \omega^2 \phi(r)^2}{4r^2 (r^2 - 1)} + \mu \left(b2(r) \phi(r)^2 + b1(r) \phi'(r)^2 \right) \right]. \tag{A.3}$$

From condition (a) of section (3) the solutions for $b1$ and $b2$ are given by,

$$\begin{aligned}
 b1(r) &= \frac{1}{2r (r^2 - 1)^2} \left((-4r^3 - 12r + \omega^2) a3(r) \right. \\
 &\quad + (r^2 - 1) (2\kappa r^4 - a6'(r)r^3 - 4\kappa r^2 + 2a3'(r)r^2 \\
 &\quad \left. + 2 (r^2 - 1) h2(r)r - 2 (r^2 - 1) j2(r)r + a6'(r)r + 2\kappa + 2a3'(r)) \right) \tag{A.4}
 \end{aligned}$$

$$\begin{aligned}
 b2(r) &= -\frac{1}{16r^2 (r^2 - 1)^4} \left((\omega^4 + 144r^3 \omega^2) a3(r) \right. \\
 &\quad + 4 (r^2 - 1) (-4r^2 f2(r) (r^2 - 1)^3 + (2r^2 g2'(r) (r^2 - 1)^2 \\
 &\quad + (\omega^2 \kappa - 2r^2 (r^2 - 1) j2''(r)) (r^2 - 1) \\
 &\quad \left. + r\omega^2 a3''(r)) (r^2 - 1) + (1 - 11r^2) \omega^2 a3'(r) \right). \tag{A.5}
 \end{aligned}$$

The boundary term coming from the original action (after adding *Gibbons-Hawking* boundary terms) are given by,

$$\begin{aligned}
 S^B &= \frac{1}{16\pi G_5} \int \frac{d^4 k}{(2\pi)^4} \left[-\frac{\phi(r)^2}{r^2} + \phi(r)^2 + r\phi'(r)\phi(r) - \frac{\phi'(r)\phi(r)}{r} \right. \\
 &\quad + \mu \left(\frac{1}{2} g2(r) \phi(r)^2 - \frac{1}{2} j2'(r) \phi(r)^2 \right. \\
 &\quad + h2(r) \phi'(r) \phi(r) - j2(r) \phi'(r) \phi(r) - \frac{1}{2} a6'(r) \phi'(r) \phi(r) \\
 &\quad + \frac{a3'(r) (\phi(r)\omega^2 + 4 (r^4 - 1) \phi'(r)) \phi(r)}{4r (r^2 - 1)^2} \\
 &\quad - \frac{a3(r) (6r\phi(r)\omega^2 + (r^2 - 1) (8r^3 + 24r - \omega^2) \phi'(r)) \phi(r)}{4r (r^2 - 1)^3} \\
 &\quad - \frac{a3(r) \phi'(r) (\phi(r)\omega^2 + 4 (r^4 - 1) \phi'(r))}{4r (r^2 - 1)^2} \\
 &\quad \left. \left. + a3(r) \phi'(r) \left(-\frac{\phi(r)\omega^2}{2r (r^2 - 1)^2} - \frac{(r^4 - 1) \phi'(r)}{r (r^2 - 1)^2} \right) \right) \right]. \tag{A.6}
 \end{aligned}$$

And the boundary terms coming from the effective action are given by,

$$\begin{aligned}
 S_{\text{eff}}^{\mathcal{B}} = \frac{1}{16\pi G_5} \int \frac{d^4 k}{(2\pi)^4} \left[\left(r - \frac{1}{r} \right) \phi(r) \phi'(r) \right. \\
 + \frac{\mu}{2r(r^2-1)^2} \left(\phi(r) (2\Gamma(r^2-1)^3 + (-a6'(r)r^3 \right. \\
 + 2a3'(r)r^2 + 2(r^2-1)h2(r)r - 2(r^2-1)j2(r)r \\
 \left. \left. + a6'(r)r + 2a3'(r)(r^2-1) + (-4r^3 - 12r + \omega^2)a3(r))\phi'(r) \right) \right]. \quad (\text{A.7})
 \end{aligned}$$

Let the form of the solution of ϕ is given by,

$$(1-r^2)^{i\beta\omega} (1 + i\beta\omega\mu F(r)) \quad (\text{A.8})$$

with

$$F(0) = 0. \quad (\text{A.9})$$

The imaginary part of the retarded Green function for original action is given by,

$$\begin{aligned}
 \frac{1}{\omega} \text{Im} \left[G_{xy,xy}^{R(\text{original})} \right] = \lim_{r \rightarrow 0} \left[-2\beta + \frac{1}{r(r^2-1)^3} \left(\mu\beta(4(r^2+3)a3(r)r^2 + (r^2-1) \right. \right. \\
 (F'(r)r^6 + a6'(r)r^4 - 3F'(r)r^4 - 2a3'(r)r^3 - 2(r^2-1)h2(r)r^2 \\
 \left. \left. + 2(r^2-1)j2(r)r^2 - a6'(r)r^2 + 3F'(r)r^2 - 2a3'(r)r - F'(r)) \right) \right] \quad (\text{A.10})
 \end{aligned}$$

and imaginary part of the retarded Green function for effective action is given by

$$\begin{aligned}
 \frac{1}{\omega} \text{Im} \left[G_{xy,xy}^{R(\text{effective})} \right] = \lim_{r \rightarrow 0} \left[-2\beta - \mu \left(\frac{1}{(r^2-1)^3} \left((r^2-1) \right. \right. \right. \\
 (2\Gamma r^4 - a6'(r)r^3 - 4\Gamma r^2 + 2a3'(r)r^2 + 2(r^2-1)h2(r)r \\
 - 2(r^2-1)j2(r)r + a6'(r)r + 2\Gamma + 2a3'(r)) - 4r(r^2+3)a3(r) \\
 \left. \left. \left. - rF'(r) + \frac{F'(r)}{r} \right) \beta \right) \right]. \quad (\text{A.11})
 \end{aligned}$$

Therefore, in low frequency limit the difference between the imaginary part of retarded Green function coming from this two boundary terms are given by ,

$$\lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{Im} \left[G_{xy,xy}^{R(\text{original})} \right] - \frac{1}{\omega} \text{Im} \left[G_{xy,xy}^{R(\text{effective})} \right] = 2\mu \beta \Gamma. \quad (\text{A.12})$$

Therefore for this general class of theory,

$$\Gamma = 0. \quad (\text{A.13})$$

The other kind of higher derivative theory one can consider is covariant derivatives acting on curvature tensors. In that case one can have a more general action like (3.3). For

this kind of action the boundary terms one gets are of the form $\phi^{(n)}\phi^{(p)}$ (here $\phi^{(n)}$ means n -th derivative of ϕ with respect to r). Using the form of ϕ given in (3.23) it can be shown that except $\phi^{(n)}\phi$ kind of terms, other boundary terms do not contribute in low frequency limit. For example, if we consider $C_n\phi^{(n)^2}$ term in the original action, the relevant boundary term which will contribute in low frequency limit is $(-1)^{(n+1)}(C_n\phi^{(n)})^{(n-1)}\phi$. One can check that though we need to add *Gibbons-Hawking* terms to make the variation of the action well defined but most of them are zero in low frequency limit. We have checked it for few nontrivial terms like, $\phi^{(3)^2}$ and $\phi^{(4)^2}$ and Γ turns out to be zero. But we expect it is true in general.

B Expressions for A^{GB}

$$\begin{aligned}
 A_1^{GB}(r, k) &= \frac{8r^2 - \omega^2 r - 8}{4r^3(r^2 - 1)} - \frac{1}{(12(r^3(r^2 - 1)^2))} ((10c_1(88r^4 - 11\omega^2 r^3 - 176r^2 + 13\omega^2 r + 88) \\
 &\quad + c_3(144r^8 - 288r^6 + 66\omega^2 r^5 + 232r^4 + 25\omega^2 r^3 - 4(3\omega^4 + 44)r^2 + 13\omega^2 r + 88) \\
 &\quad + c_2(176r^4 - 22\omega^2 r^3 - (3\omega^4 + 352)r^2 + 26\omega^2 r + 176))\mu) + O(\mu^2) \\
 A_2^{GB}(r, k) &= -\frac{3(r^2 - 1)}{r} \\
 &\quad + \frac{(10c_1(13r^2 - 11) + 2c_2(2r^4 + 17r^2 - 9) + c_3(34r^4 + 9r^2 - 8\omega^2 r - 3))\mu}{r} + O(\mu^2) \\
 A_3^{GB}(r, k) &= 4(c_2 + 4c_3)r(r^2 - 1)^2\mu + O(\mu^2) \\
 A_4^{GB}(r, k) &= -\frac{2(r^2 + 3)}{r^2} \\
 &\quad + \frac{1}{3r^2(r^2 - 1)} (2(10c_1(13r^4 + 20r^2 - 33) + c_2(26r^4 + 3\omega^2 r^3 + 40r^2 + 3\omega^2 r - 66) \\
 &\quad + c_3(90r^6 - 89r^4 + 30\omega^2 r^3 + 32r^2 + 6\omega^2 r - 33))\mu) + O(\mu^2) \\
 A_5^{GB}(r, k) &= -\frac{4(r^2 - 1)}{r} \\
 &\quad + \frac{2(20c_1(13r^2 - 11) + 2c_3(18r^4 + r^2 - 11) + c_2(52r^2 + 3\omega^2 r - 44))\mu}{3r} + O(\mu^2) \\
 A_6^{GB}(r, k) &= 8(r^2 - 1)(c_2 r^2 + 4c_3 r^2 + c_2)\mu + O(\mu^2) . \tag{B.1}
 \end{aligned}$$

C Expressions for A^W

$$\begin{aligned}
 A_1^W &= \frac{8r^2 - \omega^2 r - 8}{4r^3(r^2 - 1)} \\
 &\quad + \frac{(-360r^9 - 240r^7 + 129\omega^2 r^6 + 1560r^5 - 300\omega^2 r^4 + 8(\omega^4 - 120)r^3 + 75\omega^2)\gamma}{4(r^2 - 1)^2} + O(\mu^2) \\
 A_2^W &= -\frac{3(r^2 - 1)}{r} + r(-419r^6 + 668r^4 - 24\omega^2 r^3 + 8r^2 - 225)\mu + O(\mu^2) \\
 A_3^W &= 32r^5(r^2 - 1)^2\mu + O(\mu^2) \tag{C.1}
 \end{aligned}$$

$$\begin{aligned}
 A_4^W &= -\frac{2(r^2+3)}{r^2} - \frac{2(2045r^8 - 4185r^6 - 26\omega^2r^5 + 2140r^4 - 2\omega^2r^3 + 75r^2 - 75)\mu}{r^2-1} + O(\mu^2) \\
 A_5^W &= -\frac{4(r^2-1)}{r} - 4(r(145r^6 - 220r^4 + 2\omega^2r^3 + 75))\mu + O(\mu^2) \\
 A_6^W &= 32r^4(2r^4 - 3r^2 + 1)\mu + O(\mu^2) .
 \end{aligned}
 \tag{C.2}$$

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